

# Stability analysis of prey predator model with Holling II functional response and threshold harvesting for the predator

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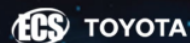
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## Stability analysis of prey predator model with Holling II functional response and threshold harvesting for the predator

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**Abstract.** This paper deals with a prey predator model with Holling response function of type II and continuous threshold harvesting in the predator population. The prey grows as a logistic model when there is no interaction with the predator. The predator is assumed decreasing due to natural death and interspecific interaction when there is no interaction with its prey. The existence of the interior equilibrium point is considered and the stability is analysed using linearization and eigenvalues methods. The phase portrait of the model is also used to determine the behaviour of populations. From the analyses of the model with harvesting we found that there exists a stable interior equilibrium point. The predator population will remain sustainable when the size of the populations are initially close enough to the equilibrium point. But when the threshold value is too high and the populations are initially quite far from the equilibrium point, then the predator population may stop being harvested at a certain time. Some numerical simulations are given to confirm the result of analysis.

### 1. Introduction

The growth rate of a certain population is not only influenced by nutrition and its biotic and abiotic environment, but it is also influenced by the other populations living together in the environment. The form of influence of one population to another population can be in the form of symbiosis, competition, predation, or another forms. Besides that, human intervention factors can also change the population growth, including harvesting and restocking activities.

The most common form of interaction between two or more populations is predation. One population acts as a predator and the other population acts as a prey. The functional response of the predator to its prey depends on characteristic of the predator and the prey. Some kinds of functional response were classified by Holling such as type I, type II, type III, type IV, and many more types. The use of Holling functional response of type II in the model of population dynamics has been widely studied [1, 2]. Holling type II functional response was also used in the prey predator model because the assumptions are quite relevant to characteristic of the populations, see for example in [3, 4].

The dynamics of prey and predator populations that involve populations as beneficial stocks, such as fish populations, have been related to the exploitation activities. The used forms of exploitation depend



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on the characteristic of the considered population and policy of the management of renewable resources. The exploitation can also be applied to the prey and the predator simultaneously [5, 6, 7] and there is also exploitation only applied to the prey population [8] or only applied to the predator population [9, 10]. The effect of harvesting can cause the population gradually becomes extinct, or it can change the value of equilibrium points.

In this study, one prey and one predator population model with Holling response function of type II is analysed. The prey population grows via logistic model when there is no interaction with the predator and the predator population decays when there is interaction between the predator and the prey populations. In the predator growth mechanism, we suppose that there is competition and intraspecific interaction. The selected harvesting is applied only to the predator. We assume that there is no harvesting when the size of predator population is still small. The population will be harvested when the size of the population reaches a certain size. The harvesting function is assumed to be continuous and bounded. The existence and stability of the positive equilibrium point are analysed using linearization and eigenvalues methods. The effects that can be caused by harvesting activities with threshold are also analysed.

## 2. Development of the prey predator model

The prey predator population model with Holling type II functional response is quite often observed by researcher. In this article, we start with a prey predator model with Holling type II functional response, such model has been proposed by Rosenzweig-MacArthur, see [11], as

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{x}{K}\right) - y \left(\frac{sx}{1+saax}\right) \\ \frac{dy}{dt} &= ey \left(\frac{sx}{1+saax}\right) - py.\end{aligned}\tag{1}$$

Particularly in [11], model (1) is then developed by considering a continuous threshold harvesting in the predator population. All parameters of the model (1) are assumed to be positive and the meaning of variables and parameters can be seen in detail in [11]. The model becomes

$$\begin{aligned}\frac{dx}{dt} &= x(1-x) - a\left(\frac{xy}{1+bx}\right) \\ \frac{dy}{dt} &= y\left(\frac{bx}{1+bx} - d\right) - H(y),\end{aligned}\tag{2}$$

where  $H(y)$  is the threshold harvesting function and given by

$$H(y) = \begin{cases} 0, & \text{if } y < y_c \\ \frac{h(y-y_c)}{h+(y-y_c)}, & \text{if } y \geq y_c. \end{cases}\tag{3}$$

Besides harvesting with constant effort and constant quota, non-smooth harvesting function [12] and impulsive harvesting and stocking [13] are also often considered. Model (2) with harvesting function (3) is then developed by considering competing and intraspecific in the predator population mechanism. Some prey predator population models with intraspecific in predator has been studied, see for example in [14, 15]. The developed prey predator model with intraspecific in the predator population is given by

$$\begin{aligned}\frac{dx}{dt} &= rx \left(1 - \frac{x}{K}\right) - y \left(\frac{sx}{1+saax}\right) \\ \frac{dy}{dt} &= ey \left(\frac{sx}{1+saax}\right) - py - cy^2.\end{aligned}\tag{4}$$

After doing normalization of the model (4), we get

$$\begin{aligned}\frac{dx}{dt} &= x(1-x) - a\left(\frac{xy}{1+bx}\right) \\ \frac{dy}{dt} &= u\left(\frac{xy}{1+bx}\right) - vy - wy^2,\end{aligned}\tag{5}$$

where  $a = \frac{sK}{re}$ ,  $b = saK$ ,  $u = \frac{seK}{r}$ ,  $v = \frac{p}{r}$ , and  $w = \frac{cK}{re}$ .

The model (5) is then applied a continuous threshold harvesting. The model becomes

$$\begin{aligned} \frac{dx}{dt} &= x(1-x) - a\left(\frac{xy}{1+bx}\right) \\ \frac{dy}{dt} &= u\left(\frac{x}{1+bx}\right) - vy - wy^2 - H(y), \end{aligned} \quad (6)$$

where the continuous threshold harvesting for the predator population  $H(y)$  follows equation (3).

### 3. Existence and stability of the equilibrium point

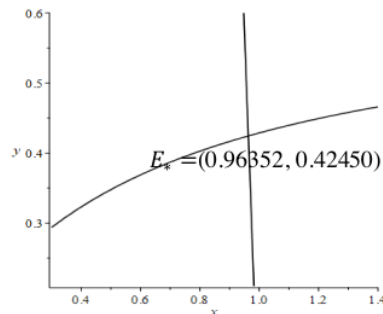
Equilibrium points of model (6) together with equation (3) and the condition  $y \geq y_c$  are found by solving the system of equations  $\frac{dx}{dt} = 0$  and  $\frac{dy}{dt} = 0$  with respect to  $x$  and  $y$  simultaneously. From  $\frac{dx}{dt} = 0$  we obtain

$$y = \frac{-bx^3 + (b-1)x^2 + x}{ax}. \quad (7)$$

Substituting (7), i.e.  $y = \frac{-bx^3 + (b-1)x^2 + x}{ax}$ , into equation  $\frac{dy}{dt} = 0$  we get

$$\begin{aligned} (1+bx) &\left(ax(-bv x^3 + (b-1)vx^2 + vx)(-bx^3 + (b-1)x^2 + (1-ay_c + ah)x) + \right. \\ &(b^2wx^6 - 2bwx^5 + wx^4 - (2b^2 + 2b)wx^3 + (2b+2)wx^2 + b^2w + 2bw + w)(-bx^3 + \\ &(b-1)x^2 + (1-ay_c + ah)x) + a^2x^2(-bhx^3 + (b-1)hx^2 + (h-ay_ch)x) \left. - \right) \\ &a^2ux^3(-bx^3 + (b-1)x^2 + (1-ay_c + ah)x) = 0 \end{aligned} \quad (8)$$

It is quite difficult to find out the roots of equation (8) analytically. We claim that under a certain condition there exists non-negative equilibrium points, namely  $E_0 = (0, y_0)$  and  $E_* = (x_*, y_*)$ , where  $y_0, x_*$ , and  $y_*$  are positive, and also the condition  $y \geq y_c$  must be satisfied. For example, suppose that the parameter values of the model are  $a = 0.5, b = 5, v = 0.2, h = 4, w = 0.3, u = 5$ , and  $y_c = 0.2$  with appropriate units. Then we have non-negative equilibrium point  $E_0 = (0, 0.16055)$  and  $E_* = (0.96352, 0.42450)$ . Because of the model is valid only when the condition  $y \geq y_c$  is satisfied, then the equilibrium point  $E_0 = (0, 0.16055)$  will not be considered since  $y_0 < y_c$ . In this model, we just consider the positive equilibrium point. The positive equilibrium point as the intersection between the two simple of isoclines is illustrated in figure 1.



**Figure 1.** Intersection between the simple isoclines  $\frac{dx}{dt} = 0$  and  $\frac{dy}{dt} = 0$ .

Analysis of stability of the equilibrium point is done via linearization method and eigenvalues of Jacobian matrix. The Jacobian matrix of the model is given by

$$J = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, \quad (9)$$

where

$a_{11} = 1 - 2x - a \frac{y(1+bx)-bxy}{(1+bx)^2} = 1 - 2x - \frac{ay}{(1+bx)^2}$ ,  $a_{12} = -\frac{ax}{1+bx}$ ,  $a_{21} = \frac{u}{(1+bx)^2}$ , and  $a_{22} = -v - 2wy - \frac{h^2}{(h+(y-y_c))^2}$ . The interior equilibrium point is denoted as  $E_* = (x_*, y_*)$ .

Substituting  $E_* = (x_*, y_*)$  into Jacobian matrix (9) to get

$$J(E_*) = \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}, \tag{10}$$

where

$$b_{11} = 1 - 2x_* - \frac{a y_*}{(1+bx_*)^2}, \quad b_{12} = -\frac{ax_*}{1+bx_*}, \quad b_{21} = \frac{u}{(1+bx_*)^2}, \quad \text{and}$$

$$b_{22} = -v - 2wy_* - \frac{h^2}{(h+(y_*-y_c))^2}.$$

From the Jacobian matrix (10) we get the characteristic equation

$$\lambda^2 + A_1\lambda + A_2 = 0, \tag{11}$$

where  $A_1 = -(b_{11} + b_{22})$  and  $A_2 = (b_{11}b_{22} - b_{21}b_{12})$ .

The equilibrium point  $E_* = (x_*, y_*)$  is asymptotically stable when the Routh-Hurwitz stability criteria from the characteristic equation (11) are satisfied, that are  $A_1 > 0$  and  $A_2 > 0$ .

#### 4. Numerical simulations

##### 4.1. Case 1. Threshold $y_c = 0.2$

Suppose that the parameter values of the model (6) are given by  $a = 0.5$ ,  $b = 5$ ,  $u = 5$ ,  $v = 0.2$ ,  $y_c = 0.2$ ,  $w = 0.3$ , and  $h = 4$  with appropriate units. We then have an interior equilibrium point  $E_* = (0.96352, 0.42450)$ . The characteristic equation associated with the equilibrium point is  $\lambda^2 + 1.45644\lambda + 0.49343 = 0$ . Therefore we have eigenvalues  $\lambda_1 = -0.92022$  and  $\lambda_2 = -0.53621$ . Some trajectories around the equilibrium point are given in figure 2, 3, and 4.

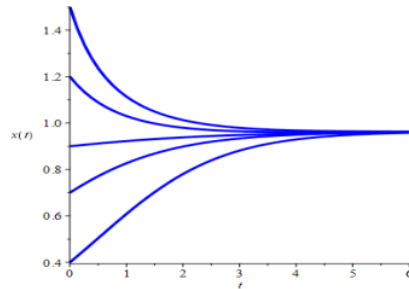


Figure 2. Some trajectories of prey population with threshold  $y_c = 0.2$ .

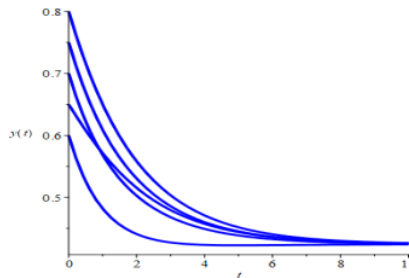
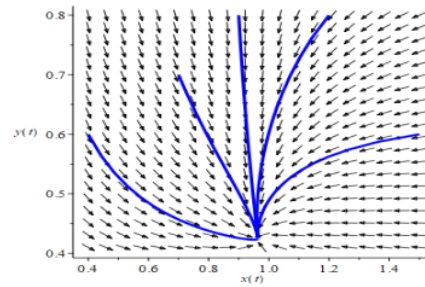


Figure 3. Some trajectories of predator population with threshold  $y_c = 0.2$ .

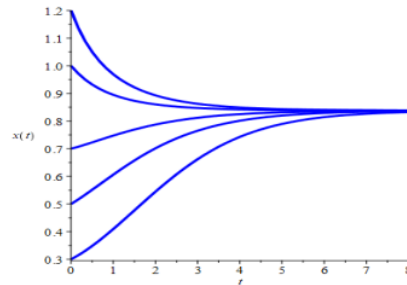


**Figure 4.** Some trajectories of prey and predator populations with threshold  $y_c = 0.2$ .

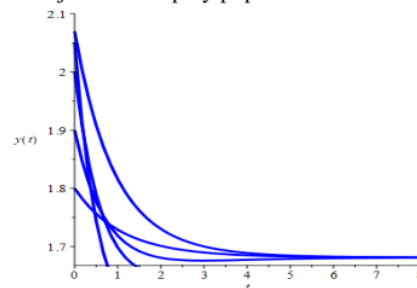
Figure 2 and 3 give illustrations that when the prey and predator populations initially close to the equilibrium point then the trajectories finally tend to the stable equilibrium point. While figure 4 illustrates that there exists a basin in the first quadrant so that the equilibrium point is globally asymptotically stable. In this case, the harvesting will not affect the existence and the stability of the equilibrium point provided that the populations are initially in the basin. The two populations will remain sustainable for a long period of time although the predator population is harvested.

**4.2. Case 2. Threshold  $y_c = 1.5$**

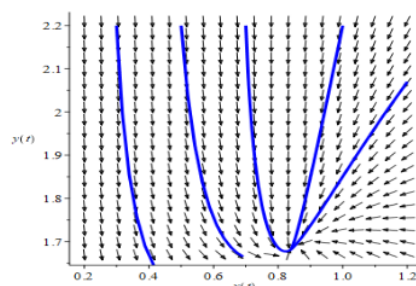
Suppose that the parameter values of the model (6) are given by  $a = 0.5$ ,  $b = 5$ ,  $u = 5$ ,  $v = 0.2$ ,  $y_c = 1.5$ ,  $w = 0.3$ , and  $h = 4$  with appropriate units. The parameter values are the same with the case 1, except for threshold value. Then we have an interior equilibrium point  $E_s = (0.79242, 2.06011)$ . The characteristic equation associated with the equilibrium point is  $\lambda^2 + 2.28450 \lambda + 1.07231 = 0$ . Therefore we have eigenvalues  $\lambda_1 = -0.66014$  and  $\lambda_2 = -1.62436$ . Some trajectories around the equilibrium point are given in figure 5, 6, and 7.



**Figure 5.** Some trajectories of prey population with threshold  $y_c = 1.5$ .



**Figure 6.** Some trajectories of predator population with threshold  $y_c = 1.5$ .



**Figure 7.** Some trajectories of prey predator populations with threshold  $y_c = 1.5$ .

Figure 5 illustrates that when the prey and the predator populations are initially close enough to the equilibrium point then the trajectories finally tend to the stable equilibrium point. While figure 6 and 7 illustrate that when the populations are initially quite far from the equilibrium point, the size of harvested population may not satisfy the threshold value. Even though the equilibrium point is stable, it is just locally stable. In this case, the harvesting will affect sustainability of the harvested population. The predator population is initially harvested, but later at one time the harvesting will be stopped when the size of predator population,  $y(t) < 1.5$ .

## 5. Conclusions

The prey predator population model with intraspecific interaction and selected continuous threshold harvesting for the predator population is analysed. Because of the model is nonlinear, it is quite complex to get the equilibrium points analytically. The model may have non-negative equilibrium points. Under a certain condition of the parameter values of the model, we may have a positive equilibrium point.

With suitable values of parameter, there exists a positive equilibrium point (case 1). From the linearization approach, the equilibrium point is locally asymptotically stable. From the phase portrait analyses of the model (figure 4), it seemingly exists a basin for the equilibrium point. This means that there exists a basin so that the equilibrium point is globally asymptotically stable. Therefore, the population will not be extinct for a long time although the predator population is harvested with a threshold value and the populations are initially close enough to the stable equilibrium point.

In case 2 with threshold value  $y_c = 1.5$ , the model still has a positive equilibrium point. The equilibrium point is locally asymptotically stable. This means that when the initial value of the populations are close enough to the equilibrium point, the trajectories will lead to the stable equilibrium point. But, when the initial population is quite far from the equilibrium point, the trajectories of populations may not lead to the stable equilibrium point. There exists a certain time so that the size of predator population will be less than the threshold value (figure 7). At that time, the predator population must be stopped from harvesting since the model does not follow the given condition any more, i.e.  $y \geq y_c$ .

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